Inflation, Uncertainty, and Investment

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Abstract

This paper investigates the effect of inflation on a firm's investments in fixed assets. When future prices are certain, inflation affects the present value of depreciation tax shields, and the impact of inflation on the choice between different lived assets is non-monotonic. Future asset price uncertainty creates a valuable switching option and benefits shorter-lived assets.

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Inflation causes prices to increase with a commensurate decrease in the value of money-fixed claims. In general, future inflation rates are uncertain and difficult to predict. Uncertainty about the inflation rate creates uncertainty about future relative goods prices. This paper investigates the effect of uncertain inflation on firms' investments in fixed assets. Classifying assets according to their durability, we consider firms' optimal investment rules and the impact of their investment decisions under different inflationary regimes.

The analysis is divided into two parts. Section 2 describes the impact of a onetime change in the anticipated level of inflation on asset choices in a world of certainty. In this case, a change in the rate of inflation affects only the money-fixed components of asset value, that is, the tax shields generated by an asset's taxable depreciation schedule. We show that when the analysis is limited in this fashion, the impact of inflation is non-monotonic. If asset prices are in equilibrium at relatively low rates of inflation and interest, then a small increase in inflation increases the relative breakeven price of a short-lived asset, which is the maximum price investors are willing to pay for the short-lived asset given a particular price of the long-lived asset. Conversely, if prices are initially in equilibrium at relatively high rates of inflation and interest, then a small increase in inflation decreases the breakeven price of a short-lived asset.

The non-monotonic effect of inflation under certainty was initially discovered by Nelson [12] and subsequently analyzed by Auerbach [2], Abel [1], and Brenner and Venezia [6]. We show by numerical analysis using actual depreciation schedules applicable in the seventies and early eighties that this "tax shield effect" is most significant for inflation and interest rates in the range of 0% to 20%. Thus the levels of inflation and

interest rates experienced in the U.S. between 1965 and 1985 were sufficient to induce the maximum degree of change in investment decisions and in relative asset prices.

However, inflation's impact is not limited to its effect on the value of depreciation tax shields. A growing body of research in the area of "real options" emphasizes that the volatility of relative prices also plays a role in optimal asset selection. In Section 3, we suppress the detailed analysis of tax shields to focus on the effects of price volatility. We model the investment in durable assets as a real option. Shorter-lived assets have a higher option value than longer-lived assets because the opportunity to switch between the alternative technologies arrives sooner. Therefore, higher uncertainty about future relative asset prices increases the breakeven price of a short-lived asset relative to a long-lived asset.

Section 4 integrates the results of Sections 2 and 3, summarizes the findings and presents our conclusions.

2. The Firm's Decision Problem under Certainty

Consider a firm that wants to purchase a machine to accomplish some task. Assume that two machines are available: a short-lived machine that has a useful life of S periods and a long-lived machine that has a useful life of L periods. The choice between the machines is based on the net present value of using each machine. Define v_{s0} as the present value of purchasing a short-lived machine on the first round and behaving optimally thereafter; $v_{\ell 0}$ is the value of purchasing a long-lived machine initially and behaving optimally thereafter. Since the purchase of the machine is an outflow v_{s0} and $v_{\ell o}$ are negative. The firm optimally picks the highest,

that is, least negative, of the two present values. The choice between the two machines is affected by inflation in two ways. First, depreciation creates tax deductions which are nominal assets. Second, price uncertainty makes the replacement cost uncertain at the time the first machine is purchased.

To analyze the firm's decision problem we make the following assumptions:

(A.1) Depreciation is tax deductible and the tax rate on corporate income is τ;

(A.2) Assets qualify for an investment tax credit, ITC, which is proportional to the purchase price of the asset;

(A.3) The firm realizes the full value of all tax shields;

(A.4) The after-tax riskless nominal interest rate equals R in each period.[1]

Also, we define K_s and K_ℓ as the purchase price of short-lived and long-lived machines respectively; d_{st} and $d_{\ell t}$ are the proportion of the assets' purchase price that can be depreciated in period t.

At the time the first machine is purchased, the scheduled depreciation tax deductions for that machine are known. Uncertainty in the first round is thus limited to uncertainty about nominal interest rates over the next S or L years. The firm can hedge such uncertainty by issuing debt with a payment schedule that matches the depreciation tax deductions. Thus, all cash flows pertaining to the first round of investment can, by suitable market transactions, be reduced to a single certain cash outflow today.

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[1]We assume a uniform term structure of interest rates for notational convenience. All of the results present in this section carry over with minor modifications to an environment in which the term structure is not (Footnote continued)

However, when future relative prices are uncertain, the risk associated with the second and subsequent investments cannot be hedged away, because the firm does not know the cost or the tax shields associated with future purchases at the time of the initial investment. The impact of inflation on machine purchase decisions can thus be partitioned into two effects: (i) the change in the value of tax shields on the first round of investment; and (ii) effects on the second and subsequent rounds of machine purchases. We analyze each of these effects separately.

In this section we assume that the value of replacing a machine S or L periods from now is constant in real terms. Thus, $(1 + R)^{-S}\ v_{sS}$, the present value of purchasing a short-lived machine S periods from now is:

$$(1 + R)^{-S}\ v_{sS} = (1+r)^{-S}\ v_{s0}, \tag{1}$$

and, $(1 + R)^{-L}\ v_{\ell L}$, the present value of purchasing a long-lived asset L periods from now is:

$$(1 + R)^{-L}\ v_{\ell L} = (1+r)^{-L}\ v_{\ell 0}, \tag{2}$$

where r is the real after-tax interest rate, which is assumed to be known and unaffected by inflation.

Since there is no uncertainty attached to subsequent rounds of investment under this formulation, and since prices of both machines grow at the same rate, the firm's choice between purchasing long or short term assets can be made once and for all. That is, without loss of value, the firm may choose a program of all-short or all-long-lived assets at the time of its initial investment. If $v_{s0} > v_{\ell 0}$ then short-lived machines will be

(Footnote continued)
uniform. An after-tax discount rate is appropriate because the firm can create additional tax deductions by borrowing: see Ruback [13].

preferred at each decision point; conversely, if $v_{s0} < v_{\ell 0}$, long-lived machines will be preferred. The net present value of buying the short-lived machine is:

$$v_{s0} = K_s[-1 + ITC + \tau \sum_{t=1}^{S} d_{st} (1+R)^{-t}] + (1+r)^{-S} v_{s0} \quad , \tag{3}$$

and the net present value of buying the long-lived machine is:

$$v_{\ell 0} = K_\ell[-1 + ITC + \tau \sum_{t=1}^{L} d_{\ell t} (1+R)^{-t}] + (1+r)^{-L} v_{\ell 0} \tag{4}$$

The first term on the right-hand side of (3) and (4) is the net after-tax cost of purchasing the machine, that is, the actual purchase price less the investment tax credit and the present value of the depreciation tax shields. The second term on the right-hand side of (3) and (4) is the present value of future machine purchases. Since (3) and (4) include the value of the future replacement decisions, v_{s0} and $v_{\ell 0}$ are the present values of an infinite replacement chain starting with the short-lived machine and the long-lived machine, respectively.

For both machines to continue in production, the purchase prices of the machines, K_s and K_ℓ, must be set to make the values of the two alternative investments equal, $v_{s0} = v_{\ell 0}$. This implies that, in equilibrium, the price of the short-lived machine, K_s, relative to the price of the long-lived machine, K_ℓ, satisfies:

$$\frac{K_s}{K_\ell} = \frac{[-1 + ITC + \tau \sum_{t=1}^{L} d_{\ell t}(1+R)^{-t}]}{[-1 + ITC + \tau \sum_{t=1}^{S} d_{st}(1+R)^{-t}]} \cdot \frac{1-(1+r)^{-S}}{1-(1+r)^{-L}} \tag{5}$$

The first ratio on the right-hand side of (5) is the ratio of the percentage net cost to the purchaser of a long-lived to a short-lived

machine. The second ratio adjusts the percentage net cost for the difference in timing of replacements - every S periods or every L periods - under the two programs.

Given our assumptions of certainty of future asset prices and a constant real interest rate, inflation affects the ratio K_s/K_ℓ solely through its impact on the nominal after tax interest rate, which in turn affects only the present value of depreciation tax shields. As the value of tax shields is diminished by higher nominal interest rates, the net costs of purchasing a short- or long-lived machine increase, but by different amounts. As a function of the nominal interest rate, the ratio of percentage net costs (the first ratio in (5)) is a single peaked function which equals one if the nominal interest rate is zero, increases, then decreases and approaches one as the nominal interest rate approaches infinity.

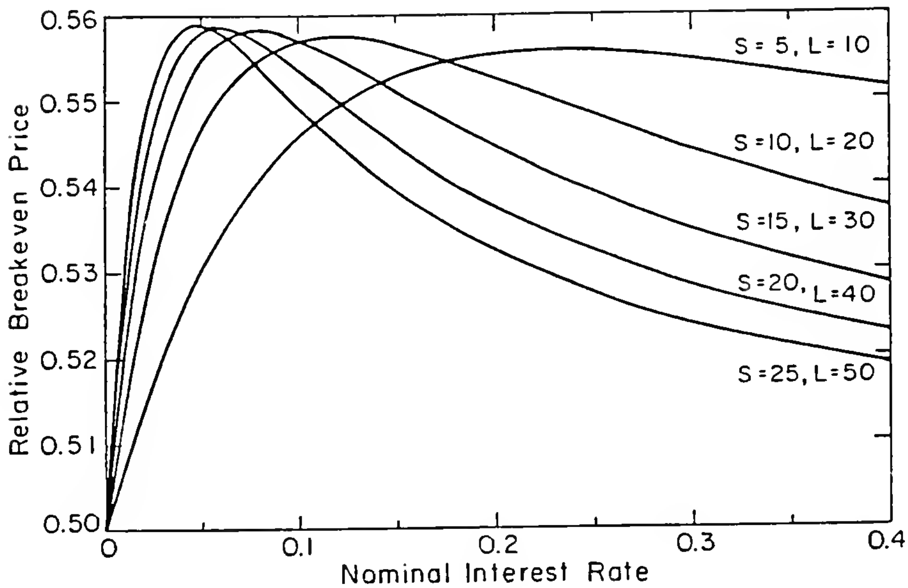
Exhibit 1 shows breakeven values of K_s/K_ℓ for various nominal interest rates and asset life comparisons. In each of our asset life comparisons, the life of the long-lived asset is twice the life of the short-lived asset. The calculations assume double-declining balance depreciation with an optimal shift to straight-line, no investment tax credit, and a zero real interest rate.

When the nominal interest rate is zero, the present value of depreciation tax deductions equals the purchase price of each asset. In this case, the breakeven price is the ratio of asset lives, one-half. Exhibit 1 shows that as the nominal interest increases, the breakeven price ratio initially rises, but at some level, a further increase in the nominal interest rate causes the breakeven price to decline. Using the comparison

Exhibit 1

Breakeven Prices of Short-Lived Assets Relative to

Long-Lived Assets for Various Nominal Interest Rates.[a]



Asset Lives		Nominal Interest Rate						
Long	Short	0%	5%	10%	15%	20%	30%	40%
10	5	.5	.531	.546	.553	.556	.555	.551
20	10	.5	.547	.557	.556	.553	.544	.537
30	15	.5	.555	.557	.551	.545	.535	.528
40	20	.5	.558	.553	.545	.538	.528	.523
50	25	.5	.559	.549	.539	.532	.524	.519

[a] A breakeven price is the maximum price that a firm would pay for a short-lived asset relative to a long-lived asset. The calculations assume double-declining depreciation with an optimal shift to straight-line, no investment tax credit, and a real interest rate of zero. In the figure, S is the life of the short-lived asset and L is the life of the long-lived asset.

between a 10-year short-lived asset and a 20-year long-lived asset, the breakeven price ratio rises from 0.5 to 0.547 as the nominal interest rate rises from zero to 5%. In other words, at a 5% nominal interest rate, purchasers are willing to buy a short-lived asset at 54.7% of the price of a long-lived asset. The breakeven price ratio continues to rise reaching a maximum of .5575 when the nominal interest equals 11.7%. As the nominal interest rate increases further, the breakeven price ratio falls; at a nominal interest rate of 20%, the breakeven price ratio is 0.553.

The most interesting thing to note in Exhibit 1 is that switches in preference for long- or short-lived assets occur at rates that are within the range of recent U.S. experience, 5% to 20%. Relatively modest changes in the nominal interest rate thus have the potential to cause substantial shifts in buyer preferences and purchasing behavior. For example, if the supply price of a 10-year asset is 55% of the price of a 20-year asset, the 20-year asset is preferred to the 10-year asset when the nominal interest rate is less than 5.8%. At nominal interest rates between 5.8% and 22.3%, the 10-year asset is preferred. However, once nominal interest rates rise above 22.3%, the 20-year asset is again preferred.

The effects of inflation on breakeven asset prices arise because the timing of depreciation tax shields differs for short- and long-lived assets. Interest rate changes thus affect the present value of the assets' respective tax shields differently. These differential effects can be eliminated by equalizing the tax service lives of assets which are substitutes.

The Economic Tax Recovery Act of 1981 introduced the Accelerated Cost Recovery System (ACRS) which equalized the tax service lives of assets

within three broad categories: light vehicles, equipment, and structures. Assets which are substitutes are likely to fall within a given category, and thus the potential for inflation-induced changes in buyer preferences based on the changing value of depreciation tax shields was substantially reduced under the ACRS.

Current proposed tax revisions recommend abandoning ACRS and adopting an "economic" depreciation schedule similar to the depreciation method used in our simulations. Some of the proposed revisions include an indexation of the book value of assets to the inflation rate. If this indexation is structured so that the present value of depreciation tax shields is independent of the rate of inflation, the differential effect of inflation on the choice between assets of varying lives that existed prior to the adoption of ACRS will not re-occur. However, to the extent that the indexation is incomplete, the effect of inflation on the choice between different-lived assets will re-occur.

3. The Impact of Uncertainty

In this section we examine the firm's optimal decision rule when future relative prices are uncertain. To suppress the detailed consideration of tax shields, define C_s and C_ℓ as the net cost including tax shields of choosing the short- or long-lived technology:

$$C_{s0} = K_s \; [-1 + ITC + \tau \sum_{t=1}^{S} \frac{d_{st}}{(1+R)^t}], \text{ and}$$

$$C_{\ell 0} = K_\ell \; [-1 + ITC + \tau \sum_{t=1}^{L} \frac{d_{\ell t}}{(1+R)^t}] \; .$$

At the time of the initial investment C_{s0} and $C_{\ell 0}$ are known, but future values of C_s and C_ℓ are uncertain. We assume the decision-maker knows the joint distribution of C_S and C_ℓ at all future points in time.

To recast the firm's problem in continuous time, we re-write (3) and (4) as:

$$v_{s0} = -C_{s0} + e^{-\alpha S} E_0(V_S) \tag{6}$$

for the short-lived asset, and

$$v_{\ell 0} = -C_{\ell 0} + e^{-\alpha L} E_0(V_L) \tag{7}$$

for the long-lived asset, where V_t is the value of the firms' replacement program at time t assuming the firm follows an optimal policy from t onward and α is the continuously compounded discount rate appropriate for the riskiness of the investment program. The second right-hand side terms in (6) and (7) represent the expected value at time 0 of the value of future replacements at the end of the first cycle.

The opportunity to switch between short- and long-lived assets is an option. For example, suppose the firm initially invests in the long-lived asset based on the breakeven relative prices derived in Section 2. If the interest rate rises such that the short-lived asset is now preferred, the investor will switch at the end of the first cycle. Compared to being committed to one technology forever, this ability to switch between assets, in general, adds value to the replacement program. The value of the switching option increases as the life of the asset decreases because the firm is allowed to switch sooner. Therefore, the option value component of the short-lived asset exceeds the option value component of the long-lived asset.

A number of recent papers focus on capital budgeting under uncertainty and on the value of real options. Contributors to this research include:

Baldwin and Meyer [3], McDonald and Siegal [10], Myers and Majd [11], Majd and Pindyck [8], and others. However, previous option-based capital budgeting models have generally made one of two quite restrictive assumptions. They assume that either the stochastic processes governing the real variables (e.g. machine prices) are stationary or that they follow geometric Brownian motion, usually with constant drift and variance parameters. These assumptions provide the highest degree of analytic tractability and may be reasonable approximations of actual price paths for some assets. However, neither of these assumptions seems perfectly consistent with our expectations about the stochastic behavior of prices of machines which are substitutes.

A <u>stationary</u> joint distribution of costs implies that the machine prices will revert to their initial distribution on the next round. Therefore, current machine prices provide no information about future machine prices. For example, if short-lived assets currently dominate long-lived assets, stationarity implies that there is no reason to anticipate that such a relationship will hold in the future. Rather, a steady state equiliLrium price relationship is always expected, although random factors may cause actual prices to deviate from the equilibrium at any point in time. Given that the two machines are substitutes, such an assumption seems reasonable if the next decision point is in the intermediate or distant future. However, the assumption of stationarity becomes less attractive as the interval between successive decisions becomes shorter.

If machine prices follow <u>geometric Brownian motion</u>, the best estimate of the future relative price is the current value adjusted for the drift. Thus machine prices are not expected to revert to their initial distrib-

ution on the next round. For example, if short-lived assets currently dominate long-lived assets and the variance and drift parameters are constant, then the current relationship would be expected to continue forever. Such an assumption about the behavior of real asset prices seems plausible in the short-run, but is implausible in the intermediate or long run because the machines are substitutes.

Intuitively, the most likely form of price behavior for two assets which are substitutes would be a combination of the extremes of a stationary distribution and geometric Brownian motion with constant parameters. Some possible processes include linear birth-and-death processes[2] and Brownian motion with state-dependent drift and variance parameters.[3] However, we know very little about the actual probabilistic behavior of prices of real substitutes, and thus do not feel comfortable selecting a specific stochastic model. Moreover, the similarity of results across all forms of "real option" models indicates that the value attached to flexibility should not depend on the specific stochastic process assumed. Therefore, we derive our results in a framework that encompasses the extremes of stationary distributions and geometric Brownian motion as well as the more realistic intermediate alternatives.

We limit our analyses to stochastic processes governing the value of the replacement program, V_t, that are stationary or Markov and that satisfy the following criteria:

$$E_0|V(t+\Delta)| = a(\Delta)\ E_0|V(t)| < e^{\alpha\Delta}\ E_0|V(t)|, \text{ for all } \Delta \text{ and } t \geq 0; \qquad (8)$$

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[2] See, for example, Baldwin [3].

[3] See, for example, Kulatilaka [7].

This condition indicates that the expectation of the absolute value of the replacement program at time $t+\Delta$, the left-hand side of (8), can be decomposed into the product of the expected absolute value of the program at time t and a function, $a(\Delta)$, of the elapsed time between t and $t+\Delta$. The function $a(\Delta)$ may also depend on the initial state variables of the process. The right-hand side inequality restricts the growth rate of the expected value of the replacement program to be less than the discount rate, α, that is used to value the process.[4] The absolute values are required because the replacement program is a sequence of costs and thus has negative value.

At the outset of its investment program, the firm selects the first in its sequence of investments by choosing the investment strategy encompassed by (6) and (7) that maximizes the value of its entire replacement program:

$$V_0 = \max_{s,\ell} \{v_{s0}, v_{\ell 0}\} .$$

This choice depends on current net cost of machines, C_s and C_ℓ, as well as on the value of the option to switch between technologies in the future. In the Appendix we prove that (i) this option component is valuable, (ii) the value of the switching option is higher for a short-lived asset than a long-lived asset, and (iii) the option value increases as the variance of future values of C_s and C_ℓ increases.

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[4]This condition is analogous to the restriction that the drift parameter be less than the discount rate when the real prices are assumed to follow geometric Brownian motion. See, for example, Venezia and Brenner [14], MacDonald and Siegal [10].

The proof has three parts. The first part shows that the optimal policy takes the form of comparing the actual cost of the short-term asset, C_s, to a breakeven price relative to the actual cost of a long-term asset. Given C_ℓ, we denote the breakeven price of the short-lived asset as $C_s^*(C_\ell)$. If the cost of the short-term asset is below $C_s^*(C_\ell)$, the short-term asset will be selected; if the cost of the short-term asset is equal to $C_S^*(C_\ell)$, the investor will be indifferent, and if the cost of the short-term asset is above $C_s^*(C_\ell)$, the long-term asset will be selected. There is one and only one breakeven $C_s^*(C_\ell)$ corresponding to each outcome of C_ℓ and vice versa. Part 2 proves that under uncertainty, a flexible series of choices (i.e. a rule that selects short or long according to the outcomes of C_s and C_ℓ instead of by a prearranged plan) increases the initial expected value of the overall investment program. This fact is used to show that the breakeven price of the short-lived asset relative to the long-lived asset is higher under uncertainty than under certainty. Finally, Part 3 of the proof shows that the effect of uncertainty is monotonic: Holding average C_s and C_ℓ fixed, an increase in the variance of C_s or C_ℓ leads to an increase in the value of the option. Thus, <u>ceteris paribus</u>, greater variability in asset prices causes the short-term asset to be selected at higher prices and more frequently than before.

In summary, the optimal selection rule determining the choice between short- and long-lived assets under uncertainty differs from the selection rule under certainty. An option value is attached to the choice of technology and the short-lived asset has a higher option value than the long-lived asset because the next opportunity to choose arrives sooner. Therefore, as uncertainty about future relative prices of short- and long-lived assets increases, so does the breakeven price of the short-lived

asset. In other words, prices of the short which under certainty would have caused it to be rejected, under uncertainty may lead to its selection because of the value of the switching option. Furthermore, a higher degree of uncertainty about future values of C_s and C_ℓ increases the option value and hence the breakeven price of the short-lived asset.

4. Conclusion

This paper considers the effects of inflation, interest rates and uncertainty on a firm's choice between assets of different lives. We first focus on the impact of inflation and interest rates on the value of nominal depreciation tax shields. We find that the effect of an inflation-induced increase in nominal interest rates on the breakeven prices of short- and long-lived assets is not monotonic. At low initial nominal interest rates, an increase in the interest rate increases the maximum price that the firm is willing to pay for the short-lived asset relative to the long-lived asset. At some point, the effect reverses so that a further increase in interest rates reduces the maximum or breakeven price of the short-lived asset. Simulation results indicate that these changes occur at rates that are within the range of recent U.S. experience, 5% to 20%.

Second, we focus on the impact of relative price uncertainty on the decision to choose a short- or long-lived asset. We find that uncertainty increases the breakeven price of shorter-lived assets: in other words, given uncertainty about future relative prices, short-lived assets will be chosen over long-lived assets at higher prices than under certainty. The change in the breakdown price occurs because both long- and short-lived assets have a valuable switching option. The option is more valuable for short-lived assets because the opportunity to switch occurs sooner. Furthermore the value of the option increases as the variability of future

costs of long- or short-lived machines increases. Thus increased price volatility causes the short-lived asset to be purchased at higher prices and more frequently than before.

To a first approximation, the two effects analyzed in this paper are independent. That is, an inflation-related increase in the nominal interest rate would cause one shift in relative breakeven prices, and a contemporaneous increase in relative price volatility would cause another. In some cases, the two effects might offset one another, making the net effect relatively small, while in others, the two effects might reinforce one another, making the net effect relatively large.

Appendix A

Proposition. Let the stochastic process governing C_S and C_ℓ be a stationary or Markov process such that

$$E_0|V(t+\Delta)| = a(\Delta)\ E_0|V(t)| < e^{\alpha\Delta}\ E_0|V(t)|, \text{ for all } \Delta \text{ and } t \geq 0; \quad (9)$$

where $a(\Delta)$ may be a constant or, at most, a function of the initial state variables and the elapsed time Δ. In this case the process $V(t)$ will have the characteristics of a real option and short-term investment will be favored.

Proof.[5]

[5]Because of space constraints, some details of the proof have been omitted. A complete version of the proof is available from the authors.

Part 1. The optimal decision rule.

Our first step is to establish the form of the optimal policy. Assume that C_{s0} and $C_{\ell 0}$ are known, and that the short term asset is preferred:

$$v_{s0} = -C_{s0} + e^{-\alpha S}E_0 V(S) > - C_{\ell 0} + e^{-\alpha L} a(L-S)E_0 V(S) = v_{\ell 0} . \quad \text{(A-1)}$$

As the cost of the short-term asset increases, the left hand side of (A-1) decreases more quickly than the right, implying that for each $C_{\ell 0}$ there is at most one C^*_{s0} that makes the investor indifferent.

As $C_{\ell 0}$ increases, $C^*_{s0}(C_{\ell 0})$ also increases. This implies that the optimal policy takes the form of a one-to-one mapping $C^*_s : C_\ell \rightarrow C_s$. Given C_ℓ, if $C_s < C^*_s$, the investor selects the short term alternative, if $C_s = C^*_s$ the investor is indifferent, and if $C_s > C^*_s$, the investor selects the long term alternative.

Part 2. The impact of uncertainty

At breakeven when the investor is indifferent, $v_{s0} = v_{\ell 0}$, so that, rearranging terms in (A-1):

$$\frac{-C^*_{s0}}{C_{\ell 0}} + 1 + \frac{E_0 V(S)}{C_{\ell 0}} [e^{-\alpha S} - e^{-\alpha L} a(L-S)] = 0 \quad \text{(A-2)}$$

where $E_0 V(S)$ denotes $E_0 \max (v_{sS}, v_{\ell S})$, the value of the program at $t = S$.

Under certainty, given the assumptions of Section 2, the value of a strategy which selects the short- or long-lived asset exclusively has the same value as any mixture of shorts and longs. Now let C_{st} and $C_{\ell t}$ be uncertain, but assume their expected values and hence the values of the all-short and all-long strategies are unchanged from the certainty case. The strategy v_{sS} consists of selecting the short asset at time S and behaving optimally thereafter. The value of v_{sS} must be at least as high as the value of an alternate feasible strategy which selects only the short- or long-term asset.

To show that v_{sS} is strictly better than an all-short (or all-long) strategy, suppose that on a given round, the realization of C_s is above its mean and the realization of C_ℓ is below its mean. In this case, selecting the long asset leads to a higher overall value than adhering to the all-short strategy. Similar reasoning applies to $v_{\ell S}$.

Since both v_{sS} and $v_{\ell S}$ are higher under uncertainty than under certainty, $E_0V(S)$, the expectation of their maximum is also higher under uncertainty. A higher $E_0V(S)$ in (A-2) implies that C^*_{s0} must rise to maintain the optimal relationship, thus the breakeven cost of the short-lived asset is higher under uncertainty than under certainty.

Part 3. Increasing variance increases C^*_{s0}.

Let the joint distribution of C_s and C_ℓ be given. Define A_s and A_ℓ as the actions of selecting the short and long asset respectively on the first round, and define F^* as the probability that the short-lived asset is selected. We can then rewrite $E_0V(S)$ in terms of A_s, A_ℓ and F^*:

$$\begin{aligned} E_0^FV(S) = &+ F^*[- E_0(C_{sS}|A_s) + e^{-\alpha S}E_SV(2S)] \\ &+(1-F^*)[-E_0(C_{\ell S}|A_\ell) + e^{-\alpha L}E_SV(S+L)], \end{aligned} \tag{A-3}$$

The superscript F on the left-hand side indicates that the expectation is taken with respect to a particular joint distribution $F(C_{sS},C_{\ell S})$. (For simplicity, in what follows time subscripts have been dropped: C_s and C_ℓ are understood to be outcomes at time S.)

Applying condition (8) and collecting terms, we may rewrite (A-3), conditioned on a particular outcome of C_ℓ:

$$E_0^F[V(S)|C_\ell] = \frac{- F^*(C_\ell)\ E_0(C_s|C_s < C_s^*(C_\ell)) + [1-F^*(C_\ell)]C_\ell}{1-e^{-\alpha L}a(L) - F^*(C_\ell)(e^{-\alpha S}a(S) - e^{-\alpha L}a(L))} \ . \tag{A-4}$$

Holding the marginal distribution of C_ℓ fixed, assume that the conditional expectation, $E(C_s|C_\ell)$, is unchanged, while the conditional variance, $\sigma^2(C_s|C_\ell)$, increases. A <u>feasible</u> strategy under the new, more dispersed distribution is to set the breakeven point C_s^* such that the probability F* of selecting the short-term asset is the same. With higher dispersion and a fixed F*, the conditional expectation $E_0(C_s|C_s<C_s^*(C_\ell))$ decreases strictly. It is the only quantity on the right-hand side of (A-4) to change, and because it is negative implies an increase in the value of the overall expression. Since the <u>optimal</u> policy maximizes the right hand side of (A-4), its value must be at least as large as the value of this feasible strategy.

Given C_ℓ, an increase in the variance of C_s increases the conditional expectation, $E_0[V(S)|C_\ell]$. The unconditional expectation $E_0V(S)$ is a probability weighted sum of the conditional expectations and hence increases as well. As $E_0V(S)$ rises, C_{s0}^* must increase to maintain the optimal relationship in (A-2). A higher variance of C_s thus increases the breakeven price of the short term asset relative to the long.

The same method may be used to show that increases in the variance of C_ℓ also lead to higher breakeven prices for the short-term asset. Finally, using (A-2) and (A-4), it can be shown that the frequency with which the short-term asset is selected goes up with the variance of C_s or C_ℓ.

References

1. Abel, Andrew B., "Taxes, Inflation and the Durability of Capital," Journal of Political Economy, 89 (June 1981), pp. 548-560.

2. Auerbach, Alan J., "Inflation and the Choice of Asset Life," Journal of Political Economy, 87 (June 1979), pp. 621-638.

3. Baldwin, Carliss Y., "Optimal Sequential Investment When Capital is Not Readily Reversible," Journal of Finance, 37 (June, 1982) pp. 763-782.

4. __________ and Richard F. Meyer, "Liquidity Preference Under Uncertainty: A Model of Dynamic Investment in Illiquid Assets," Journal of Financial Economics, 7 (December, 1979) pp. 347-374.

5. __________ and Richard S. Ruback, "Inflation, Discrete Replacement and the Choice of Asset Lives," MIT Sloan School of Management Working Paper No. 1335-82, June 1982.

6. Brenner, Menachem and Itzak Venezia, "The Effects of Inflation and Taxes on Growth Investments and Replacement Policies," Journal of Finance, 38 (December 1983) pp. 1519-1528.

7. Kulatilaka, Nalim, "The Value of Flexibility," mimeo, November 1985.

8. Majd, Saman and Robert S. Pindyck, "Time to Build, Option Value and Investment Decisions," mimeo, revised November 1985.

9. Mason, Scott and Robert C. Merton, "The Role of Contingent Claims Analysis in Corporate Finance," in Recent Advances in Corporate Finance, E. I. Altman and M. G. Subrahmanyam, eds., Richard D. Irwin, Homewood Il., 1985.

10. McDonald, Robert and Daniel Siegal, "The Value of Waiting to Invest," Quarterly Journal of Economics, 1986, Forthcoming.

11. Myers, Stewart C. and Saman Majd, "Calculating Abandonment Value Using Option Pricing Theory," MIT Sloan School of Management Working Paper No. 1462-83, August 1983.

12. Nelson, Charles R., "Inflation and Capital Budgeting," Journal of Finance, 31 (June 1976), 923-932.

13. Ruback, Richard S., "Calculating the Market Value of Riskless Cash Flows," Journal of Financial Economics, 1986, Forthcoming.

14. Venezia, Itzhak, and Menachem Brenner, "The Optimal Duration of Growth Investments and Search," The Journal of Business, Volume 52 (1979), pp. 393-407.

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